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A METHOD OF STABILIZING AUTOMATIC REGULATION SYSTEMS"

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Summary: The article describes the employment of a method of conditional stability, in order to improve and stabilize characteristics of automatic regulation systems.

Considered are an electronic voltage regulator for a generator with stepped up frequency, and a power synchro^{no-servo} follower system. The adaptation of circuits for which the provisions of conditional stability are fulfilled, made it possible, in both cases, to secure quicker-acting systems than with ordinary methods of stabilization and flexible feedbacks.

The work in question considers the utilization of the phenomenon of conditional stability for the stabilization and improvement of the operating qualities of regulating devices. As is known, the occurrence of conditional stability is such, that the given regulation system has, in its open state at certain frequencies, an amplification coefficient larger than unity, with a phase angle displacement of the output voltage in relation to the input voltage of 180° , ^{and} remains stable. The Nyquist curve for this system has a characteristic "beak". The possibility of a conditional stability was mentioned for the first time in the work of Peterson, Kreer, and Wers. However, this occurrence was not utilized in practice in developed systems of automatic regulations.

Until recently the opinion was spread that conditional stability was more of theoretical interest, and would hardly be employed in practical systems. By the two examples, below,

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the utilization of conditional stability to stabilize a system of regulation and to improve its operating qualities is shown. The first example examines the stability of an electronic voltage regulator for a generator with stepped up frequency of 800 ops and power of 2 kW.

Fig. 1 illustrates a diagram ^{of} for an electronic voltage regulator. The measuring element of the regulator consists of a nonlinear bridge, three arms of which have fixed resistances, while in the fourth arm there is a diode 4D2 with a tungsten cathode (having a stable emission) whose filament is fed by the regulated voltage. The diode resistance depends on the filament current magnitude. During a voltage fluctuation of the generator, a voltage appears at the output of the measuring bridge whose polarity and magnitude depend on the direction and amount of displacement of the generator voltage from the assigned value, corresponding to the balance of the non-linear bridge. The voltage from the output of the measuring element is applied to the grid of the tube Π_3 (6H7), which is connected in the phase bridge circuit.

With a voltage fluctuation on the grid of ~~the~~ tube Π_3 , its equivalent resistance changes too, which results in a voltage phase displacement on the grids of the thyratrons, relative to the voltage on the anodes of the thyratrons Π_4 and Π_5 (Π_4 - 213). The rectified voltage, whose value depends on the voltage deviation of the generator, is applied to the excitation winding of the generator. The entire regulator is supplied by the generator voltage at a frequency of 800 ops.

^{a block}
The skeleton diagram of the regulator is illustrated in Fig. 2 and, as was demonstrated by ^{the} conducted tests, can be

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represented in the first approximation in the form of three links:
 first link - measuring element, second link - amplifier and
 power elements, third link - generator. All three links can
 be replaced by equivalent inertia elements described by operators
 of the form $\frac{K_i}{1 - pT_i}$ with time-constants T_1, T_2, T_3 , and ampli-
 fication coefficients K_1, K_2, K_3 . Inasmuch as all regulator
 elements are elements of directed operations, i. e., the
 operations in each successive link of the regulator do not
 influence the preceding link, then it is sufficient, in order
 to obtain the general equation of the system in an open state,
 to cross-multiply the operators, corresponding to each link.
 As a result we obtain

$$U_{out} = U_{in} \frac{K_{12}}{1 - pT_1 - pT_2 - pT_3} \quad (1)$$

where T_1, T_2, T_3 are time-constants of the separate regulator
 links, $K_{12} = K_1, K_2, K_3$ - overall coefficient of amplification,
 equal to the products of the amplification coefficients of sepa-
 rate links, p - differential operator. In a closed circuit
 of regulation

$$U_{in} = U_{out} \quad (2)$$

Substituting (2) in (1), we obtain, after corresponding
 transformations, the characteristic equation

$$1 - pT_1 - pT_2 - pT_3 + p^2T_1T_2 + p^2T_1T_3 + p^2T_2T_3 - p^3T_1T_2T_3 = 0 \quad (3)$$

where

$$K_{12} = K_1 K_2 K_3 \quad (4)$$

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Equation (5) is the characteristic equation, corresponding to the differential equation of the regulation system in question. Setting up ^{Hurwitz} ~~Stability~~ condition for stability and substituting values obtained experimentally for T_1 , T_2 , and T_3 ($T_1 = 0.1$ sec, $T_2 = 0.01$ sec, $T_3 = 0.35$ sec.), we shall find the limiting coefficient of amplification of the system at which stable operation of the regulator is still possible without the introduction of anti-hunt devices:

$$1 + K_{lim} \leq 52 \quad (5)$$

Or after substitution of values for T_1 , T_2 , and T_3 :

$$1 + K_{lim} \leq 52 \quad (6)$$

The actual amplification coefficient is equal to 600, therefore one has to expect that in the absence of anti-hunt devices the regulator will be unstable. This is confirmed by the oscillogram in Fig. 3, which illustrates that the regulator ^{produces} ~~performs~~ undamped oscillations.

To achieve stable operation of the regulator it is necessary to introduce in the regulation system an anti-hunt element, which, taking into consideration the requirements presented to the regulator, should be simple. Besides, it is desirable that the absence of sustained oscillations is not attained at the expense of a considerable decrease of the amplification coefficient, or at the expense of considerable decrease in the speed of regulation. In the given case, while investigating the characteristics of the system, it is expedient to use Nyquist's ^{or} method of amplitude-phase characteristics, inasmuch as this shows graphically the influence of one or another anti-hunt device

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on the regulation stability. As is known, for plotting the amplitude-phase characteristics it is necessary to substitute $p = j\omega$ in the expression for the operator of the open system $K = f(p)$ and to lay out the curve $K = f(j\omega)$ with ω changing from 0 to ∞ (the second branch for ω changing from 0 to ∞ will represent a reflected image of the first, relative to the real axis in the plane $f(j\omega)$). If, in addition, the point $(1, 0)$ lies inside of the obtained curve, then the system will be unstable; if outside of the curve - it will be stable. From (1) the operator of the open system of regulation equals:

$$K = \frac{U_{out}}{U_{in}} \text{ etc.} \quad (7)$$

The amplitude phase characteristic for an open system, described by equation (7), with the above indicated parameters, is illustrated in Fig. 4 (curve 1).

Inasmuch as the point $(1, 0)$ lies inside of the curve, the regulator is unstable. The natural frequency of the sustained oscillations of such a system corresponds, approximately to a frequency at which the characteristic intersects with the positive axis x . In the considered case, $\omega \approx 40$ which conforms quite well with the experiment (Fig. 3),

There are two basic methods for clearing the oscillations.

The first and most perfected method consists of inserting derivatives from the regulated parameter into the regulation circuit. In practice, with ordinary systems one usually is limited to the first derivative. The second method consists of changing the adduced time constants, at the expense of introducing the induction into the regulation circuit a flexible feedback. This method of eliminating sustained-oscillation is not as good as

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the first one, since the time of the transitional process usually increases. The illustrated ~~block~~ ^{block} diagram of a regulator in Fig. 2 gives, practically, only small possibilities of introducing anti-hunt devices with the stipulation that resulting circuit complications are negligible. One of the reasons for this is the impossibility for constructive considerations of introducing the feedback voltage into the measuring element of the regulator ~~in order~~ ^{in order} to utilize its amplification coefficient for the feedback voltage as well. Therefore, a more expedient regulator ~~scheme~~ with an anti-hunt element is illustrated in Fig. 5, where element IV is an element producing a voltage at the output proportionally derived from the input voltage.

The considered schemes for element IV give, practically, a distorted derivative with an inertia characterized by the time constant T.

Fig. 6 illustrates the circuit for element IV. The sum of the voltages taken from the non-linear bridge and from the resistance R_1 is applied to the input of the amplifier element. The value of the resistance R_1 should be a few times larger than the resistance introduced by the bridge, in order that the circuit $R_1 - C_1$ can be considered a link of directed operations. The voltage, taken from the output of element IV is equal, according to Fig. 6, to

$$U_{out} = - \frac{R_1 C_1}{1 + p T_4} U_{in} \quad \text{Insert eq. P. 38} \quad (9)$$

where

$$T_4 = R_1 C_1$$

With a parallel connection of element IV and element I the resulting voltage at the output of both elements, relative to the voltage of the input - U_{in} , expressed:

$$U'_{out} = - \left(\text{etc} \right) U_{in} \quad \text{Insert eq. P. 38} \quad (10)$$

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From (10), it follows that with supplementary undistorted amplification of the voltage taken off the differentiating circuit by K_2 times and with equality of time constants $T_1 = T_4$

$$U'_{out} = K_1 U_{in} \quad (11)$$

i. e., in this case, with the help of the parallel connection of the differentiating link, the time constant of the inertia element can be reduced to zero. However, the introduction of supplementary amplification of the voltage taken from element IV, would considerably complicate the regulator circuit; meanwhile, in the absence of such an amplification, the introduction of element IV cannot have significant influence on increasing the stability of the system, since the inertia element has a relatively large amplification coefficient ($K_1 = 15$). The curve in Fig. 7 illustrates the amplitude phase characteristic of the measuring element. Curve II - amplitude phase characteristic of elements I and II in parallel. The equation for the amplification coefficient of the system with the presence of element IV is:

$$K_{system} = K_1 K_2 \quad (12)$$

Corresponding with this equation, the amplitude-phase characteristic of the entire regulator is illustrated in Fig. 4 (curve II). As is seen, the introduction of circuit IV, though the system approaches somewhat a stable condition, it, nevertheless, does not lead to a full elimination of the sustained oscillations, as confirmed also by the oscillogram in Fig. 3 where the sustained oscillations of the system were taken with circuit IV in the regulation circuit. Since the above introduced stabilization method was found to be insufficiently effective for the given regulator, another method was adopted.

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Inasmuch as the inertia and differentiating elements are equivalent to certain electric systems with inductances, capacitances, and resistances, one can select a combination of elements which would be equivalent to the resonance conditions for the currents in the electrical circuit for the natural frequency of oscillation of the system. For this, it is apparently necessary that in the expression for the amplification coefficient of the system in a complex form (after substituting $p = \frac{\omega}{\lambda}$) the imaginary part approaches zero at $\lambda = \lambda_0$, where λ_0 is the natural frequency of oscillation of the system. In order to give the resonance curve a sufficiently sharp form, it is also necessary that the real part be sufficiently small.

As shown by the calculations, a series connection of the two inertia links of directed operations, shunted by a differential element, as in Fig. 9, with certain values of the parameters (T_1 , T_5 , T_4 , and K_1), is equivalent to the condition of resonance for the currents in the electric circuit at $\omega = \omega_0$.

In addition, the phase of the amplification coefficient vector changes abruptly toward the leading side, as $\frac{\omega}{\lambda}$ approaches ω_0 . The result is that point (1, 0) is not within the amplitude-phase characteristic, and the system becomes conditionally-stabilized. The analytical calculation of the parameters T_4 and T_5 for given values of T_1 and K_1 is, generally, difficult. It is expedient, therefore, for every considered system, to select parameters T_4 and T_5 by means of several ^{approximate} calculations. The ~~calculation~~ ^{approximation} is facilitated by the fact that the values T_4 and T_5 are not of a critical nature, and can lie ^{within} a fairly wide range. For the considered system the following values were chosen:

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$$T_4 = 0.5 \text{ sec.}, T_5 = 0.1 \text{ sec.}$$

The amplitude-phase characteristic of the measuring element, in the presence of only the supplementary inertia element with time-constant T_5 , is indicated in Fig. 7 (curve III). The amplitude-phase characteristic of the regulator on the whole is determined by the expression

$$\text{Insert eq. 1 to} \quad (13)$$

and indicated in Fig. 4 (curve III). As can be seen, the introduction of only one supplementary inertia link with the time-constant $T_5 = 0.1 \text{ sec.}$ aggravated, as was to be expected, the stability conditions of the system. The oscillogram of the sustained oscillation of the system corresponding to curve III in Fig. 4, is illustrated in Fig. 10.

Curve IV in Fig. 7 corresponds to the amplitude-phase characteristic of the measuring element with the introduction of element IV, in addition to the supplementary inertia element. The expression for the operator of the system in an open condition will be in this case:

$$\text{Insert eq. P. 40} \quad (14)$$

Curve IV in Fig. 4 corresponds to the amplitude-phase characteristic of the system, according to equation (14) with $p = j\omega$. As can be seen, in this case point (1, j0) proved to be outside of the amplitude-phase characteristic, i. e., the regulator will be stable (case of conditional stability).

The oscillogram in Fig. 11, corresponding to the connection and disconnection of full load on the generator, shows, that the regulator functions with full stability. The time of the transitional process is set for 0.1 sec.

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An interesting conclusion from the above indicated analysis is the possibility of obtaining stable operation of the regulator by means of a distorted derivative and a supplementary inertia link, producing a sharp phase change of the vector of the amplification coefficient toward the leading side, for a frequency approaching the natural frequency of oscillation of the system.

This method guarantees a large amplification coefficient at low frequencies, and makes it possible, consequently, to ~~obtain~~ ^{obtain} a quicker-acting system than with ordinary stabilization methods through flexible feedbacks. Thus, the method of conditional stability contrasts favorably with the stabilization methods using flexible feedbacks.

As the second example, where the conditional stability is utilized for improving the quality of regulation, let us examine briefly the ~~synchrono-follower~~ ^{servo} system with amplidyne control, whose equivalent diagram in an open condition is illustrated in Fig. 12. The first element-phase-amplifier - can be analyzed roughly as an inertia element with time-constant T_1 and amplification coefficient K_1 . The amplidyne, with its full compensation, can be approximately represented in the form of two series combined elements of directed operations, corresponding to the control circuit, including the electronic scheme and circuits of the quadrature axis of the amplidyne with an overall amplification coefficient of K_{23} , and the time constants T_2 and T_3 (time-constant T_2 is determined together with the electronic tube, in whose anode circuit is connected the control winding of the amplidyne). The fourth element represents the driving motor, and is substituted for, in the first approximation, by an inertia element with the time-constant T_4 and amplification coefficient K_4 (we shall disregard the inductance of the armature circuit).

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The last, fifth, element is an integrating one. The operator of the system in an open state is expressed by

$$\text{Insert eq. P. 42} \quad (15)$$

The numerical values of parameters for the system in question are the following: $K = K_1 \cdot K_{23} \cdot K_4 \cdot K_5 = 1375 \cdot 30 \cdot 1.5 \cdot 0.002 = 124$;

$$T_1 = 0.03 \text{ sec}, T_2 = 0.015 \text{ sec}, T_3 = 0.03 \text{ sec}, T_4 = 0.09 \text{ sec}.$$

Substituting in (15) $p = j\omega$, we shall construct the amplitude-phase characteristic for the considered system. The amplitude-phase characteristic is shown in Fig. 13 (curve I). Since the point (1,0) lies inside of the curve, the system is unstable. To stabilize the system a flexible feedback is utilized from the output of the motor (a voltage is taken off, proportional to the angular velocity of the motor) through the differentiating element (as in Fig. 6) to the input of element II. The system of elements, with flexible feedback, is illustrated in Fig. 14. The equation for differentiating element VI will be:

$$U_{05} = \text{etc. Insert eq. P. 42} \quad (16)$$

The equation for the elements encompassing the flexible feedback, according to Fig. 14, will be

$$\omega = -U_1 \frac{\text{Insert eq. P. 42}}{K_{REL} K_{05} \text{ etc}} \quad (17)$$

where $K_{05} = K_{23} K_4 K_6$, and K_{REL} is the coefficient, characterizing the relative value of voltage, taken from resistance R in Fig. 6, utilized for stabilization. The expression for the operator of the system in an open condition with flexible feedback, encompassing elements II, III and IV, has the form:

$$\text{Insert eq. P. 42} \quad (18)$$

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The amplitude-phase characteristic, corresponding to equation (18) at $K_{REL} = 0.2$, $K_{OS} = 24$ and $T_6 = 0.5$ sec, is illustrated in Fig. 13 (curve II). As the drawing shows, the system, at the given value of $K_{REL} = 0.2$ becomes stabilized. In order to increase the margin of stability, it is necessary to increase somewhat the value of coefficient K_{REL} . If in place of the differentiating element of Fig. 6 we substitute a flexible feedback element, whose circuit is illustrated in Fig. 15, the amplitude-phase characteristics of the system will change. If, in the circuit of Fig. 15, we assume that the value of resistance R_2 is large in comparison with R_1 , we ^{obtain} ~~receive~~ roughly:

$$U_{OS} = -\omega \frac{K_{REL} \text{ etc}}{\text{etc}} \quad (19)$$

where

Insert equations 19A, page 12

The overall operator of the system with the presence of the element corresponding to Fig. 15 in the flexible feedback circuit is expressed:

Insert eq P44

(20)

and the amplitude-phase characteristic corresponding to it for $K_{REL} = 0.2$ is illustrated in Fig. 16 (curve I). For comparison, an amplitude-phase characteristic (curve II) is indicated there with an ordinary layout for the differentiating element in the flexible feedback circuit (Fig. 6). As can be seen, changing the

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layout of the flexible feedback circuit ^{brought conditional} ~~caused the system to~~
~~stability to the system~~
~~become conditionally stable~~. The difference of curve I from
 curve II is due to the large amplification coefficient of the
 system at small frequencies, and, as is known, a system ^{such as} ~~this~~
 will be a quicker-acting ~~one~~. Actually, if we calculate, for
 instance, the amplitude of the error for a closed system during
 a variation in the input parameter (angle) according to "sine"
 law with an amplitude of 15° and a period of 4 sec., then, for
 a modified system of flexible feedback (Fig. 15) it will be
 equal to $\sim 2^\circ$, and for an ordinary system (Fig. 6) $\sim 4^\circ$, i. e.,
 twice as large. In regard to the part of the curve in the fre-
 quency region which characterizes a stable system, it remains
 approximately unchanged.

Therefore, in the second example, the operation of the
 system in the area of conditional stability permitted an
 increase in the rapidity of action of the system. The two
 above illustrated ~~ex~~ ^{am} ~~amples~~ of regulation systems are taken
 from experience. The indicated systems operate ~~entirely~~
 normally and do not ~~display~~ ^{show} any operating ~~deficiencies~~ ^{signal deficiencies}. However, one
 has to take into consideration that, in contrast to other ordinary
 systems, during the utilization of conditional stability, the
 decrease of the amplification coefficient below ~~the~~ fixed limit
 can produce sustained oscillations. The stabilization method
 for automatic regulation systems employing conditional stabi-
 lity cannot be called a universal method, adaptable for all
 systems; however, it permits, in many cases, ^(achievement of) ~~to achieve~~ a stable
 system of regulation while preserving its inherent rapid action.

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Fig. 1 Principle schematic diagram of an electronic voltage regulator

Fig. 2 ~~Skeleton~~ ^{Block} diagram of the regulator:
I-Measuring element, II-amplifying power elements, III-generator

U_{in}	T_1	T_2	T_3	U_{out}
	I	II	III	

Fig. 3 Oscillogram of sustained oscillation of a thyatron regulator without anti-hunt devices

Fig. 4 Amplitude-phase characteristics of an open system of regulation:

- I - without anti-hunt devices,
- II - with the presence of one differentiating element,
- III - with the presence of one supplementary inertia element,
- IV - with the presence of a differentiating and one supplementary inertia elements.

Fig. 5 ~~Skeleton~~ ^{Block} diagram of a regulator with anti-hunt element.

Fig. 6 Scheme of a differentiating element

Fig. 7 Amplitude-phase characteristics of the measuring element of the regulator:

- I - without anti-hunt devices
- II - with the presence of one differentiating element.
- III - with the presence of one supplementary inertia element
- IV - with the presence of a differentiating and one inertia element

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Fig. 8 Oscillogram of sustained oscillations of regulator with introduction into the circuit of an anti-hunt circuit conforming to Fig. 5

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Fig. 9 ~~Skeleton~~ ^{Block} diagram of the measuring element of the regulator with the presence of differentiating and supplementary inertia elements

 T_1 T_5 T_4

IV

P. 40

Fig. 10 Oscillogram of sustained oscillations of a system with one supplementary inertia element

P. 40

Fig. 11 Oscillogram, corresponding ^{to} ~~with~~ connection and disconnection of full load on the generator

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Fig. 12 ~~Skeleton~~ ^{Block} diagram of a synchro-~~servo~~ ^{no-servo} system with amplitude control, in open condition:

I - phase-amplifier, II and III amplidyne (approximately characterized by two elements of directed operation), IV - driving motor (approximated by an inertia element), V - integrating element.

 K_1, T_1 K_2, T_2 K_3, T_3 K_4, T_4

I

II

III

IV

V

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Fig. 13 Amplitude-phase characteristic of a synchro-~~servo~~ ^{no-servo} system with amplidyne control, in an open condition:

I - without flexible feedback.

II - with flexible feedback

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Fig. 14 ~~Block~~ ^{Block} diagram of flexible feedback encompassing an amplidyne (elements II and III) and driving motor (element IV). ~~accomplished by the differentiating element.~~ ^{through the differentiating element} P. 43

Fig. 15 Circuit of modified differentiating element P. 43

Fig. 16 Amplitude-phase characteristic of synchro-~~follower~~ ^{servo} system with amplidyne regulation in an open condition
 I - with flexible feedback, with modified differentiating element:
 II - with flexible feedback, with ordinary differentiating element. P. 43

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